


## Counting Elements in a subset using a Venn Diagram (Section 6.2)

Hyperlinks are shown in blue, download the [cdf player](#) from the Wolfram Alpha website to view the [Wolfram Alpha interactive demonstrations](#). When you have downloaded the cdf player, click on this symbol  to view the demonstration.

### The Inclusion-Exclusion Principle

**Definition** For any finite set,  $S$ , we let  $n(S)$  denote the number of objects in  $S$ .

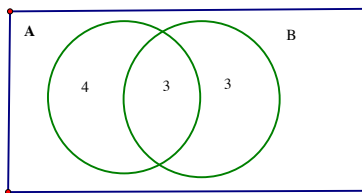
**The Inclusion Exclusion Principle** If  $A$  and  $B$  are sets, Then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

**Example** Check that the Principle is true for the following sets:

$$A = \{1, 2, 3, 4, 5, 6, 7\}, \quad B = \{5, 6, 7, 8, 9, 10\}$$

We can use a Venn diagram showing the number of elements in each basic region to display how the numbers in each set are distributed among its parts.



Thus we can sometimes use the inclusion-exclusion principle either as an algebraic or a geometric tool to solve a problem.

**Example** Let  $A$  and  $B$  be sets, such that  $n(A) = 10$  and  $n(B) = 12$  and  $n(A \cup B) = 15$ , then how many elements are in the set  $A \cap B$ ?

**Example** Let  $A$  and  $B$  be sets, such that  $n(A \cup B) = 20$ ,  $n(B) = 10$  and  $n(A \cap B) = 5$ , then how many elements are in the set  $A$ ? (Solve this using both methods: algebra and a Venn diagram)

**Example** A survey of a group of students, revealed that 60 of them liked at least one of the cereals, Frosted Flakes or Lucky Charms. If 50 of them liked Frosted Flakes and 46 of them liked Lucky Charms,  
(a) How many of them liked both cereals?

(b) Draw a Venn diagram showing the results of the survey.

(c) How many students liked Frosted Flakes but did not like Lucky Charms?

**Note** that if two sets  $A$  and  $B$  do not intersect, then  $n(A \cap B) = 0$  and hence  $n(A \cup B) = n(A) + n(B)$ . Now apply this to a set and its complement to get

$$n(A) + n(A') = n(U)$$

where  $U$  is the universal set.

**Example** A survey of 70 students revealed that 64 of them liked to learn visually. How many of them did not like to learn visually?

**Example** 68 students were interviewed about their music preferences. 66 of them liked at least one of the music types, Rap, Classical and Eighties. How many didn't like any of the above music types?

**Example** (A combination of the two principles) In a survey of 70 students on Movie preferences, the students were asked whether they liked the movies "The Breakfast Club" and "Ferris Bueller's Day Off"., (All students had seen both movies and the only options for answers were like/dislike), 50 of the students said they liked "The Breakfast Club" and 25 of them said they didn't like "Ferris Bueller's Day Off". All students liked at least one of the movies.

(a) How many students said they liked both movies?

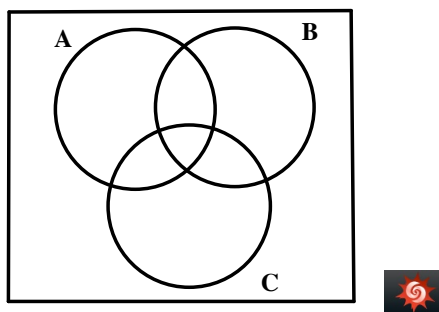
(b) Display the survey results on a Venn diagram.

**Example** In a survey of a group of 70 movie-goers, 62 liked the movie “Catching Fire”, 42 liked the movie “Divergent” and 39 liked both movies.

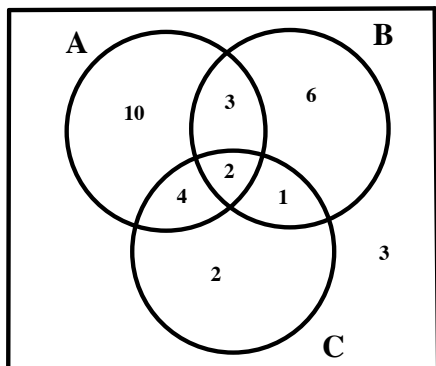
(a) Represent this information on a Venn Diagram.

(b) Use the Venn diagram to find how many of those surveyed did not like either movie.

**Venn Diagrams of 3 sets.** A Venn diagram of 3 sets divides the universal set into 8 non-overlapping regions. We can sometimes use partial information about numbers in some of the regions to derive information about numbers in other regions or other sets.



**Example** The following Venn diagram shows the number of elements in each region for the sets  $A$ ,  $B$  and  $C$  which are subsets of the universal set  $U$ .



Find the number of elements in each of the following sets:

- (a)  $A \cap B \cap C$
- (b)  $B'$
- (c)  $A \cap B$
- (d)  $C$
- (e)  $B \cup C$

**Example** In a survey of a group of 68 Finite Math students, 62 liked the movie “The Fault in our Stars”, 42 liked the movie “The Spectacular Now” and 55 liked the movie “The Perks of Being a Wallflower”. 32 of them liked all 3 movies, 39 of them liked both “The Fault in Our Stars” and “The Spectacular Now”, 35 of them liked both “The Spectacular Now” and “The Perks of Being a Wallflower” and 49 of them liked both “The Fault in Our Stars” and “The Perks of Being a Wallflower”. Represent this information on a Venn Diagram.

**Example** In a survey of a group of 68 Finite Math students (Spring 2006), 50 said they liked Frosted Flakes, 49 said they liked Cheerios and 46 said they liked Lucky Charms. 27 said they liked all three, 39 said they liked Frosted Flakes and Cheerios, 33 said they liked Cheerios and Lucky Charms and 36 said they liked Frosted Flakes and Lucky Charms. Represent this information on a Venn Diagram. How many didn't like any of the cereals mentioned?

**Example** The results of a survey of 68 Finite Math students(Spring 2006) on learning preferences were as follows: 64 liked to learn visually, 50 liked learning through listening and 36 liked learning Kinesthetically. 21 liked using all three channels, 47 liked to learn visually and through listening, 35 liked to learn both visually and kinesthetically, 21 liked to learn through listening and kinesthetically. How many preferred only visual learning?

### Old Exam questions for Review

**1** In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

- (a) 8                    (b) 10                    (c) 9                    (d) 12                    (e) 11

**2** Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

- (a) 10                    (b) 15                    (c) 4                    (d) 0                    (e) 2